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REVIEWS

Control of Chaos: Methods and Applications. II. Applications[1]

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Abstract—Reviewed were the problems and methods for control of chaos, which in the last decade was the subject of intensive studies. Consideration was given to their application in various scientific fields such as mechanics (control of pendulums, beams, plates, friction), physics (control of turbulence, lasers, chaos in plasma, and propagation of the dipole domains), chemistry, biology, ecology, economics, and medicine, as well as in various branches of engineering such as mechanical systems (control of vibroformers, microcantilevers, cranes, and vessels), spacecraft, electrical and electronic systems, communication systems, information systems, and chemical and processing industries (stirring of fluid flows and processing of free-flowing materials)).

1. INTRODUCTION

In the first years after the penetration of the concept of deterministic chaos into the scientific literature, chaotic behavior was regarded as an exotic phenomenon which might be of interest only as a mathematical speculation and would never be encountered in practice. Later on, however, the possibility of chaotic dynamics was discovered in numerous systems in mechanics, communication, laser and radio physics [10, 12, 16, 18, 19], chemistry and biochemistry [46], biology [55], economics [47, 124, 144], and medicine.

Yet further development highlighted a number of applications where chaotic modes may appear—sometimes as harmful, sometimes as useful. Moreover, entire classes of problems that are of practical importance arose where one has to control a nonlinear system by reducing or, on the contrary, increasing the degree of its chaoticity. Methods for solving these problems also were actively developed. The main of them were described in the first part [6] of the present review whose second part is devoted to their applications.

More than 300 papers devoted to various applications of the methods for control of chaotic processes were published in the peer-reviewed journals between 1997 and 2002. The questions of chaos control are actively discussed in scientific and technical fields such as physics of turbulent processes, laser physics and optics, physics of plasma, molecular and quantum physics, mechanics, chemistry and electrochemistry, biology and ecology, economics and finances, medicine, mechanical engineering, electrical engineering and chemical industry, traffic control, or communication and information systems. It is appropriate to decompose the applied works on chaos into scientific and technical (engineering) applications.

The works on engineering applications demonstrate the use of chaos and the methods for control of chaotic systems in particular practical problems or at least show their feasibility. The scientific

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applications (in physics, chemistry, or biology), on the contrary, are vectored to the control theory and methods for discovering new properties and regularities in behavior of physical (chemical, biological) systems, rather than to particular applications. They often make use of simplistic model descriptions of the systems under study. At the same time, the smallness requirement or other constraints on the class of admissible control actions play an important role. Introduction of these (explicit or implicit) constraints is aimed at elucidating the internal properties inherent in the system itself and not forced on it by a strong control action. The scientific and technical applications are discussed below, respectively, in Sections 2 and 3. The miscellaneous applications are discussed in Section 4.

2. SCIENTIFIC APPLICATIONS

2.1. Mechanics

Control of pendulums, beams, and plates. The pendulum represents the simplest class of mechanical systems featuring complex dynamics. The pendulum systems can manifest essentially "nonlinear" behavior such as multistability, bifurcations, or chaos. Simplicity and obviousness of physical experiments make the pendulum attractive both for research and tutorial purposes [33, 98, 118, 169].

Motion of the simple pendulum with friction, as well as of many other models of nonlinear oscillators with one degree of freedom, is known to be able to become chaotic if excited by a harmonic force of a sufficient amplitude. Some works examined the problem posed by S.W. Shaw in 1989: "How much does increase the amplitude of a periodic exciting force for which chaos is not yet observed, provided that the form of the exciting force can be varied?" An inverse pendulum provided with stops and excited by horizontal oscillations of the suspension axis was studied in [118]. The optimal profile of the exciting force guaranteeing (by the Mel'nikov criterion) the absence of chaos for the greatest amplitude was determined analytically, and the greatest amplitude was shown to exceed the corresponding amplitude of for harmonic excitation by the factor of two. Introduction of a feedback results in further extension of the zone of nonchaotic modes [120]. Similar results were obtained for the Duffing and Helmholtz oscillators [119].

H.K. Chen [50] investigated the dynamics of the two-gimbal gyroscope with nonlinear (cubic) damping under external harmonic action caused by vertical vibration of the base. The following model of the gyroscope nutation motion was used:

$$I_1\ddot{\theta} + \frac{\beta^2(1-\cos\theta)^2}{I_1\sin\theta^3} + M_d(\dot{\theta}) - F_g l\sin\theta = F_g\bar{l}\sin\omega t\sin\theta, \tag{1}$$

where θ is the nutation angle; $M_d(\dot{\theta}) = D_1\dot{\theta} + D_2\dot{\theta}^3$ is the torque of dissipative forces; D_1 and D_2 are parameters; F_g is gravity; l is the distance from the base to the gyroscope center of mass; $\bar{l}$ and ω are, respectively, the amplitude and frequency of external action; the parameter $\beta = I_3\Omega$; I_1 and I_3 are the gyroscope polar and equatorial moments of inertia; and Ω is the rotor velocity relative to the main axis. Sufficient equilibrium stability conditions were obtained using the Lyapunov method, and thorough numerical studies were carried out. The passage to chaotic nutations with increase of base vibration amplitude was shown by means of the bifurcation diagrams. Further, consideration was given to the problem of control, transformation of chaotic system motion into periodic. With that end in view, the potentialities of the open-loop control (action of a constant or periodic torque), time-delayed feedback control (Pyragas algorithm) [6, Section 4.5]), and adaptive control were examined. In the last case, it was assumed that the parameter β in (1) is adjusted (modified) by the integral of the mismatch between the current and desirable values of the nutation angle and its derivative. Modeling demonstrated feasibility of chaos suppression by the above methods of control. Chen [50] also considered the problem of *synchronization* of the chaotic systems of this kind.

Conditions for excitation and suppression of chaos by an external action were studied also for the models of mechanical systems such as bars and beams [43, 86], plates [49], impact systems [119, 173], chains of oscillators connected in series by elastic links [41]. It was proposed [172, 173] to accelerate the transients by intentional excitation of the chaotic modes in control systems. As was shown by the examples of bouncing ball and double pendulum, chaotization of motion of the nonlinear system enables one to reduce the time of controlled passage to the given periodic trajectory.

Control of friction. "Sliding" (low velocity) motion of the mechanical system may be of stick-slip (in particular, chaotic) nature caused by interaction of the static and kinematic frictional forces. From the practical point of view, it is advisable to control the system so as to give rise to smooth motion instead of chaotic. This control is important for the micromechanical devices such as the computer CD drives where stick-slip motions can arise upon start and stop. It is also important for the powerful drives of the artillery mounts and the telescopes. Friction forces are usually controlled chemically by means of liquid lubricants. Another approach [65, 155] relies on mechanical control pursuing a twofold purpose of (1) providing smooth motion at low velocities and (2) reducing frictional forces. A model of a system which has, in addition to the macroscopic degree of freedom, that is, position of the sliding body, an internal degree of freedom describing the lubricant state was used in [155]. On the basis of linearization of the Poincaré map and the modal control method, a control algorithm was proposed subject to the assumption that the entire four-dimensional system state vector is measurable and the control action is a normal force. The need for restoring system dynamics and, in particular, dynamics of the internal degrees of freedom in the area of the frictional contact, is a disadvantage of the approach of [155].

Two algorithms for stabilization of unstable continuously sliding states of an oscillator with dry friction were proposed in [65]. They rely on the macroscopic equations of system motion and the time-delayed feedback (method of Pyragas). Elastic deformation is used as the measured variable, and either the velocity of sliding or the normal force is used as the control action. Both methods were shown to be able of driving motion from stick-slip to continuous, and the velocity control was shown to be more precise than the load control. The reader is referred to [131] for an experimental corroboration of the possibility of suppressing or creating a chaotic mode when controlling the normal force by the algorithm of episodic (pulse) proportional feedback (the OPF-algorithm [6, Section 4.4]).

2.2. Physics

Control of turbulence. Description and control of turbulence already over an entire century remains one of the main physical problems [75]. The infinite-dimensional description of the turbulent flow as a solution to the Navier–Stokes partial equation is known to be often reducible to the finite-dimensional description. If the dimensionality of the flow attractor in the phase space is relatively small, then the turbulent flow may be regarded as chaotic, and the methods of chaos control can be applied to it. The Taylor–Couette liquid flow between two rotating concentric cylinders exemplifies such flows.

Experimental control of dynamics of the chaotic structures arising in the Taylor vortex flow with globoid (hourglass) geometry is described in [174]. This flow is a variant of the Taylor–Couette system. In the experiment, the internal cylinder was rotated by a computer-controlled stepper. Use was made of a water-glycerol mix with 1.5 volume percent of the Kalliroscope suspension added for visualization. An increase in the rotation velocity leads to a greater Reynolds number $R = 2\pi fad/\nu$, where f is the rotation frequency, a is the globoid radius at the central part, d is the gap at the center, and ν is the kinematic viscosity. For $R > R_{ps}$, where R_{ps} is the critical Reynolds number for which phase slip occurs, pairs of vortices arise: first, periodic, then chaotic. The intervals I_n between phase slips were measured by a TV camera. Control was exercised by varying the reduced Reynolds number $\varepsilon = (R/R_{ps}) - 1 = (f/f_{ps}) - 1$ by means of the algorithm

$$\delta\varepsilon_{n+1} = K(I_n - I_F) + R\delta\varepsilon_n, \tag{2}$$

where $\delta\varepsilon_n = \varepsilon_n - \overline{\varepsilon}$ and $\overline{\varepsilon}$ corresponds to the periodic unstable motion with the interval I_F between the phase bands. The control signal was applied only if the condition $|\delta\varepsilon_{n+1}| < 0.01$ was satisfied, which corresponds to the generalized OPF-algorithm (or a special case of the recurrent algorithm described in [70] and also [6, Section 4.4]). The parameters I_F, K, and R of the control law were selected experimentally. It was established that it suffices to vary ε at most by 2% in order to suppress chaos for $\overline{\varepsilon} = 0.417$ (corresponding to the chaotic process).

Control of lasers. Suppression of the chaotic or so-called *multimode* behavior of lasers is discussed in one of the first publications on control of chaotic systems [153]. This work presented experimental data on feedback leading which enabled an appreciable (by an order of magnitude) improvement in the radiation power owing to more powerful pumping. Scores of papers on chaos control in lasers and optical devices appeared in journals in 1997–2002. Recent studies were devoted mostly to the methods of open-loop control [6, Section 4.1] and the time-delayed feedback (Pyragas method [6, Section 4.5]). The effect of time-delayed feedback on the dynamics of laser with modulation of losses was studied in [14]. Experimental suppression of the Lorenz-like instability by a time-delayed feedback in ammonium lasers was described in [64]. The methods of control by open loop and time-delayed feedback for the CO_2-lasers with modulation of losses and also for the doped Nd fiber laser were compared in [81] which predicted numerically that the stability domain of the lasers of class B would be expanded (shift of the period doubling bifurcation) by exercising control on the basis of models with two degrees of freedom. Analytical facts were corroborated by simulations and experiments.

Control of chaos in plasma. Successful control of chaos in the so-called Pierce diode was described in [23, 74, 148]. The Pierce diode is one of the simplest models for studying plasma stability. Oscillations can occur both in the virtual kinetic cathode and hydrodynamic plasma. The OGY method was used in [148] to stabilize cycles of periods 1 and 2. The signal of time-delayed feedback by measurements of the space charge density at a fixed space point was used in [23] to suppress chaos by modulating the difference of potentials between the input and output diode grids. The results of [23, 148] can be used to drive a device to the mode of stable microwave oscillations.

The findings on the multimode feedback control for the magnetohydrodynamic modes and on using feedback in plasma diagnostics were generalized in [158]. The studies were aimed at developing methods for experimental determination of models of plasma turbulence dynamics for both better understanding of the transfer processes and more reliable design of controllers. A new feedback-based method of experimental structure of nonlinear dynamic models plasma turbulence was described. The results of this work were corroborated experimentally for the Columbia Linear Machine and may be extended to the fusion machines.

Interaction of laser radiation and plasma, which plays an important role in controlled thermonuclear fusion, was studied in [154]. It is noted that on the whole the interaction is very complicated and its mathematical model has not yet been established, but two types of phenomena are known to be observed at it. One lies in occurrence of stable soliton-like structures, and the other, in occurrence of extremely unstable chaotic processes. It is assumed that these phenomena can be studied separately. The authors of [154] examine the means for reducing the chaotic process to periodic oscillations or to a steady state by the two types of control, respectively: open-loop periodic variation of a system parameter or the so-called "proportional pulse control." The paper focuses on the following model:

$$\begin{cases} \dot{x} = -gx - b_1(x+z)y^2 \\ \dot{y} = -g_0 y + b_2(x^2 - z^2)y \\ \dot{z} = -g_0 z + b_3(x+z)y^2, \end{cases}$$

where x, y, and z are, respectively, the dimensionless field amplitudes for anti-Stokes, pump, and Stokes modes and g, g_0, b_1, b_2, and b_3 are the parameters. Differentiation is carried out with respect to the spatial coordinate along the direction of wave propagation. The open-loop control lies in varying the parameter g_0 according to $g_0(t) = \bar{g}_0 - a\cos\omega_0 t$. For $\bar{g}_0 = 1$ and $a = 0$ (no control), chaotic behavior is observed. The system process can be made periodic by an appropriate choice of the amplitude a and frequency ω_0 of modulation of the parameter g_0. Proportional pulse control lies in discontinuous variation of the state variables at certain time instants.

The results of experimental studies of the chaotic processes in n-conductivity germanium oscillistors can be found in [92]. Behavior of Kadomtsev–Nedopasov instability in electron-hole plasma at temperatures 77° and 300° K under the action of external electrical and magnetic fields was studied. Pictures of the space-time profile of chaotic processes were obtained from the measurements at various points of the specimens. Bifurcation diagrams showing the boundaries of domains with double period, quasiperiodicity, chaoticity, and intermittence were obtained. Several attractors having each its own dimension and energy response were shown to be feasible simultaneously in the specimens for certain conditions. Study of semiconductor plasma subjected to an external harmonic action was continued in [27] which presented some experimental dependences such as fractal, d_f, and Kaplan–Yorke, d_{K-Y}, dimensions vs. electric field intensity. This work also considered synchronization-induced input amplification in a certain amplitude-frequency domain.

Synchronization of the chaotic space-times structures (patterns) in the spatially distributed models of semiconductor heterostructures by means of the time-delayed feedback was considered in [156] which compared control with the diagonal feedback matrix, global control, and their combination. Consideration was given to two models of semiconductor nanostructures that are of current interest: superlattice and two-barrier diode with resonance tunneling. Quality of control in these systems was shown [29] to improve by several orders of magnitude owing to suitable filters and couplings based on the Floquet eigenmodes of the unstable orbits. For the mechanism resulting in a better control on the basis of phase synchronization of the desired process and that in the control loop, an explanation was given.

Control of the dipole domains. As was demonstrated in [157], propagation of the dipole domains in the GaAs/AlAs-superlattice can be controlled by an external high-frequency field. The doped GaAs/AlAs-superlattice manifests negative conductivity, which leads to propagation of the dipole domains. Depending on the frequency of external action, various, including chaotic, modes of domain propagation occur. Frequency locking was shown to be realizable by maintaining the external field frequency within a certain range which extends with the amplitudes of high-frequency voltage. Outside this range, quasiperiodic and chaotic oscillations occur.

2.3. Chemistry

Chaotic oscillations in chemical reactions were discovered in the 1970's first by modeling and then by experiments for the brusselator models under external action, coupled brusselators, and the Belousov–Zhabotinsky reaction. The methods of chaos control in chemical reactions were proposed in [80, 145, 146]. For example, the authors of [146] described application of the proportional (map-based, OPF) control algorithm to stabilization of the periodic mode of Belousov–Zhabotinsky reaction. The objective of control is formulated either as the attainment of the reaction steady mode, that is, suppression of chaotic oscillations, or as excitation of an oscillatory or even chaotic mode. For example, chaotic behavior is desirable for combustion processes because it enhances agitation of the air-fuel mix and, consequently, acceleration of the process [57]. Since chaos leads to a better stirring, reaction often is more uniform and, therefore, the product is less polluted. Earlier results on prediction and control for the steady modes of chemical reactions based on linearization of the simple three-variable autocatalator model

$$\begin{cases} \dot{\alpha} = \mu(\kappa + \gamma) - \alpha\beta^2 - \alpha \\ \sigma\dot{\beta} = \alpha\beta^2 + \alpha - \beta \\ \delta\dot{\gamma} = \beta - \gamma, \end{cases} \tag{3}$$

where α, β, and γ are dimensionless concentrations and σ, δ, μ, and κ are dimensionless model parameters (for $\sigma = 0.015$, $\delta = 1$, $\mu = 0.301$, and $\kappa = 2.5$ the system manifested chaotic behavior), were generalized in [147]. It deserves noting that the opinion of its authors that the proposed algorithms—the so-called time series-based methods of control—do not use models of the controlled process does not represent the facts. The proposed algorithms, as well as the OGY method [134, 6, Section 4.4], are special cases of the algorithm of parametric adaptive control, and the model of controlled process actually exists in the system as an adaptive model obtained from the current observations.

Resonant chaos control by light in a chemiluminescent Belousov–Zhabotinsky reaction with catalysis by a cerium–ruthenium mix was proposed in [82]. Control was exercised by varying the light flow. A chemical reactor with continuous stirring (CSTR) was used for the experiments. The light flow had the form of a sequence of "rectangular" pulses. Control based on the Montanator model with seven variables succeeded in stabilizing several unstable periodic orbits (processes) with different periods.[2] Feasibility of stabilizing the orbits of periods 2 and 4 of the Belousov–Zhabotinsky reaction in the above reactor was shown in [163]. Experiments on using the open-loop control of complex—including chaotic—oscillatory electrochemical processes were described in [138] which demonstrated that by choosing an appropriate frequency of external voltage one can not only make a chaotic process periodic, but also affect dynamics of regular oscillations. In the experiments, the amplitude of input voltage did not exceed the rated value more than by 5 %, which is indicative of the possibility of practical application of this "resonance" method of control. The possibility of eliminating chaos in coupled electrochemical oscillators by open-loop control and time-delayed feedback was shown in [105, 107, 136, 137]. Facts about experimental control of chaos in electrochemical dissolution of copper in phosphoric acid by means of a neural network can be found in [106]. Electrodissociation of the nickel-based electrodes in sulphuric acid was discussed in [107]. Behavior of a single chaotic oscillator and an array of sixty-four interrelated chaotic oscillators was studied experimentally. External harmonic (program) action and also feedback were fed into the control electrode. The work demonstrated that the stages of nonsynchronized chaos, intermittent chaotic clusters, stable chaotic clusters, stable periodic clusters, periodic synchronous process, and the steady mode occur successively with increase in the feedback factor.

Chemical reaction-based control of continuous crystallization of dibasic lead phosphite was considered in [110, 111]. The initial mathematical model was rearranged in a logistic equation underlying a modified OGY algorithm to stabilize the cycle of period 2. Periodic measurement of a liquid flow was used to transform chaotic oscillations into cyclic oscillations of period 6. The algorithm of feedback control was compared in [111] with the so-called *derandomization algorithm* which lies in periodic external action on the system. The feedback algorithm was shown to be advisable because it features higher precision.

[2] By the period of a discrete-time process is meant the number of steps between repetitions of its values. For example, the process x_k, $k = 1, 2, \ldots$ is said to have period 2 if $x_{k+2} = x_k$, but $x_{k+1} \neq x_k$ for all k.

2.4. Biology and Ecology

It seems that the question of chaos suppression in ecological system was first considered in [1, 2] as early as in 1985 for a system of fourth order describing the dynamics of an aquatic ecosystem consisting of two kinds of mictroalgae and two kinds of zooplankters. It was shown that chaotic oscillations can be transformed into periodic by means of a weak periodic action on the system parameter; at that, the value of the parameter never leaves the chaoticity domain. Although it was noted in [167] that the presence of chaos in natural populations is questionable, even a simple linear feedback (OPF-algorithm) or open-loop control was demonstrated to stabilize the population dynamics rather quickly and be good for monitoring parasites, insects, or other biological species. The authors of [162] applied the OGY method and the predictive control to the one-dimensional discrete Ricker model describing the dynamics of one-species population and to the continuous Scheffer model of plankton dynamics of the third order.

The possibility of controlling population of the red flour beetle *Tribolium castaneum* was considered in [59] which developed a mathematical model in the form of the system

$$\begin{cases} L_k = bA_{k-1}\exp(-c_{EL}L_{k-1} - c_{EA}A_{k-1}) \\ P_k = L_{k-1}(1-\mu_L) \\ A_t = P_{k-1}\exp(-c_{PA}A_{k-1}) + A_{k-1}(1-\mu_A) \end{cases}$$

of three deterministic difference equations describing the population dynamics (the so-called LPA-model), where L_k is the number of fed larva; P_k is the number of nonfed larva, chrysalises, and immature insects; and A_k is the number of individuals that are ready for multiplication. The remaining values are parameters. The exponential factors in the right-hand sides describe cannibalism among the insects and have the sense of the densities of probabilities that some individuals eat up the others. The discrete time instants $k = 0, 1, \ldots$ correspond to the instants of real time with the two-week interval. This model was verified experimentally. In qualitative terms, it describes well the phenomena characteristic of population such as occurrence of steady states and periodic, quasiperiodic, and chaotic oscillations depending on the system parameters. Minor variations in the number of mature individuals were shown in the laboratory environment to be usable for controlling substantial fluctuations in the number of insects [59].

A simple algorithm to support the transient chaos on the basis of the discrete model

$$y_{k+1} = F(y_k, u_k),$$

where y_k is the kth maximum (or minimum) of the scalar output, was obtained in [60] (the point map gives a reasonable approximation for systems with strong dissipation). This method was applied to (1) elimination of voltage collapse in power systems, (2) preservation of species in ecology, and (3) elimination of undesirable bursting phenomena in chemical reactions.

2.5. Economics

Admittedly, the dynamics of many economic systems obey the nonlinear models; the systems may manifest chaotic behavior [124]. The problems of controlling such systems are quite realistic only at the microeconomic level. Here, suppression of chaos is the reasonable aim of control, which leads to higher predictability of the business cycles. Control of business cycle using a continuous-time version of the Metsler model was considered in [70]. An adaptive control algorithm leading to a satisfactory chaos suppression was obtained on the basis of the speed-gradient method. The authors of [89] considered control of chaos for the model of a microeconomic system describing two competing companies using different investment strategies. Both companies are assumed to occupy dominating positions in the same market sector. the Behrens–Feichtinger model

$$\begin{cases} x_{k+1} = (1-\alpha)x_k + a\left(1 + e^{-c(x_k - y_k)}\right)^{-1} \\ y_{k+1} = (1-\beta)y_k + b\left(1 + e^{-c(x_k - y_k)}\right)^{-1}, \end{cases} \tag{4}$$

where α, β, $0 < \alpha$, and $\beta < 1$ are the rates of decrease in the cost of goods sold in the absence of investments, the parameters a and b characterize efficiency or scale of investments, and c is the so-called elasticity measure of the investment policy, is used to describe situations of both companies trying to exercise management simultaneously in the presence of an external disrurbance. The “parasitic” oscillations about the periodic orbit that destruct the anticipated stabilization effect were shown to be plausible in the case of competition in management. The OGY model was used to control chaos. Chaos was demonstrated to be suppressible by an appropriate choice of parameters, provided that only one company exercises management. If both companies try to manage the market simultaneously, then one often fails to eliminate chaos. In [90] such results were obtained for an algorithm with time-delayed feedback. Interestingly, Eqs. (4) first appeared as a model of the armament race between two countries with asymmetrical policies in the field of armaments. The authors of [91] considered a model example of controlling the Behrens–Feichtinger system by the Pyragas method in an attempt to compare the processes in the chaotic model of [124] with real economic time series.

2.6. Medicine

Study and treatment of cardiac arrythmia was among the most thrilling and promising among the early applications of chaos control [76]. Design of the high-speed feedback pacemakers seemed to be a radically new approach in cardiology. However, behavior of the human heart turned out to be more complicated than it seemed a decade ago. Several models and methods for controlling the chaotic processes of cardiac activity have been proposed since. A method based on the one-step linear time-delayed feedback was proposed in [44] for suppressing the pathological rhythm of period 2.

A method of controlling the distributed processes of wave instability in cardiac tissues on the basis of time-delayed feedback was proposed in [150]. The first experimental results on using control in clinical practice seem to be related with treatment of human atrial fibrillation [61]. Twenty-five patients were involved in the studies. A four-polar catheter with electrodes was introduced through the patient’s femoral vein and placed in the lateral right atrium. Fibrillation was excited by rapid pacing of 50 Hz frequency during one to two seconds. Then, the *learning phase* ensued during which the (unstable) periodic orbit to be stabilized was determined. Control was aimed at stabilizing the interbeat intervals by an algorithm of the OGY type. As was mentioned in [61], nine of the twenty-five patients manifested perfect chaos control (36 %), partial control was observed for ten patients (40 %), and no control was observed for the remaining six patients (6 %).

Another medical problem, reduction of the level of chaotic oscillations in seasonal epidemics, was considered in [79]. The process was described by the classical epidemiological model where the rate of vaccination is used as control. Chaos was shown to be eliminable if using a constant and sufficiently high rate. The authors assert that reduction, rather than complete elimination, of chaoticity is the most suitable aim of control. The paper also demonstrated effectiveness of the PID laws of control and their robustness to inaccuracies in the model parameters.

As was noted in [88], which is of a review nature, the methods of the chaos theory can be of use to cope with the difficulties of controlling the blood sugar at diabetes, select correctly individual treatment procedures, and even automatize this process in future. This work considered plausible methods of modeling chaotic oscillations of sugar content and discussed possible ways to reducing their swing.

The experimental results on chaos control in the animal neuron networks were presented in [52]. In particular, suppression of epileptiform activity was observed in sections of the hippocamp under the action of direct small-amplitude current. The author also reported on experiments of exciting stochastic resonance in the mammal brains under the action of modulated electrical field and studies of a feedback algorithm for suppression of oscillations in the activity of the neural networks that are characteristic of epileptic seizures.

Behavior model of a collection of nephrons—functional structural units of kidneys—was discussed in [164] which presented a system of differential equations describing pressures and liquid flows in an individual nephron. Oscillatory, including chaotic, modes and conditions for their origination and synchronization are analyzed using this model. The paper also presented the results of experimental studies which are in good agreement with the calculated data.

3. TECHNICAL APPLICATIONS

3.1. Mechanical Systems

Irregular oscillations in mechanical systems caused by rotation of unbalanced rotors, vibrations in spatially extended structures, and so on occur in many technical applications. Suppression of undesirable oscillations is a characteristic objective of control. Problems of this sort are often tackled by the methods of the linear control theory. According to the existing information, there are successful examples of using nonlinear control. Some of them are described below.

Control of vibroformers. Aluminium is produced from aluminum in electrolyzers. Carbonic blocks consisting of petroleum coke, recycled spent anodes, and binder pitch are used as anodes. Their production is an important stage of the process. Coke and wastes are crushed to the desired size, mixed, and heated. Pitch is added to the heated mass and mixed until paste is obtained which is used to make blocks of mass 1 to 1.2 ton. This operation is done by the vibration compactors (vibroformers). The blocks are baked at the temperature 1100° C. The quality of anodes has a significant impact on the efficiency of the reduction process. Low density reduces its efficiency, and high permeability increases emission of carbon in air and carbon dioxide into the reductive cell. Additionally, defects in anodes increase their electrical resistance. To avoid these drawbacks, anodes should be made of a uniform high-density material. Vibrational compacting is much more efficient than simple compression. It makes for better stirring of the material, production of better blocks, and also eliminates air bubbles that reduce the strength of the anode blocks. As is known from practical work, regular oscillations of vibroformers lead to low-grade anodes. However, too irregular oscillations of great amplitude may result in excessive jumps of the pusher and, therefore, destruction of anode. A method of vibroformer control supporting the given degree of irregularity by irregular switching between several periodic modes of the installation was proposed in [140]. The vibroformer is a kind of the impact oscillator obeying the bouncing ball model:

$$-0.5g(t-t_k)^2 + v_k(t-t_k)^2 + a\sin(\omega t_k) = a\sin(\omega t), \tag{5}$$

$$v_{k+1} = \alpha\big(v_k - g(t_{k+1}-t_k)\big) + a(1+\alpha)\,\omega\cos(\omega t_{k+1}), \tag{6}$$

where $x(t)$ is the pusher position and $y(t)$ is the vertical displacement of the vibroformer platform which is regarded as harmonic, $y(t) = a\sin(\omega t)$. The reduction coefficient $\alpha \in (0,1]$ describes the impact energy dissipation and depends on the characteristics of the pusher and material of anode ($\alpha = 1$ corresponds to absolutely elastic collision). The time instants t_k when $x = y$ are those of impact. The pusher velocity immediately after t_k is denoted by v_k (it is assumed for simplicity that the impact contact is instantaneous). Equation (5) is solved in t, and as a result the instant

t_{k+1} of the next impact is determined from v_k. Then, the velocity v_{k+1} after the next impact is established from (6).

System (5), (6) is known to exhibit chaotic behavior within a wide range of variations of parameters. Since both regular and uncontrolled behaviors are equally undesirable, it was suggested [140] to identify and stabilize some periodic motions embedded in the chaotic mode and then pass irregularly from one to another with the aim of improving anode quality. It was proposed to stabilize unstable q-periodic motion by a linear law of control based on the measurements of pusher velocity and the predicted instant of the next impact. It was proposed to use the frequency of vibroformer rotation as the control variable. To accelerate the mode-to-mode passage, the earlier method of targeting [139, 140] is used.

Control of microcantilevers in atomic force microscopes. Dynamics of microcantilevers used in the atomic force microscopes were studied and a method for their control was proposed in [35, 36]. Microcantilever is set in oscillatory motion by a sinusoidal input, its displacement being measured by an optical system. Dynamics of forced motion are investigated by the Mel'nikov method which allows one to determine in the space of system physical parameters a domain where chaotic motion is possible. Then, determined is the Mel'nikov function for closed-loop system which depends on the parameters of the PD-controller. As the result, the parameter values are determined for which chaotic motion is impossible.

Stabilization crane oscillations. Pendulation suppression of a shipboard crane was discussed in [104]. Chaotic process with a dominating frequency lying near the natural system oscillations was considered as a disturbance. Their effect was examined. A fuzzy controller was proposed to suppress them. The cable length was used as a control variable. Studies showed that the swing of oscillations was substantially reduced as compared with the uncontrolled system.

Stabilization of ship oscillations. Roll motion of a flooded ship was considered in [130]. Great amount of water inside the hull gives rise to complicated coupled oscillations of the ship and liquid in it that are similar to oscillations of the coupled oscillators. The picture becomes even more complicated because of the quasiperiodic external disturbances. The model of forth order was used to describe the dynamics of roll in waves. In their preceding works, the authors of [130] relied on the numerical and laboratory studies to demonstrate feasibility of complex chaotic oscillations of great amplitude. In this paper they posed the problem of reducing the system to regular small-amplitude oscillations. This problem was solved by the Pyragas method of time-delayed feedback. To this end, terms proportional to the differences between current and delayed values of the angular velocities of ship roll and inclination of water in it are introduced in the right-hand sides of system equations. It was shown that the chaotic process can be reduced to a small-amplitude periodic one by an appropriate choice of the delay time and the feedback coefficients.

Suppression of chaotic oscillations of tachometer. Behavior of a mechanical tachometer subjected to additional vibrations along the rotation axis was studied in [78]. Vibrations of the base obey the harmonic oscillations $A \sin \omega t$. Presented was a mathematical model of the system. Its characteristics were studied by various analytical and numerical methods. Bifurcation diagrams demonstrating that with growth in the vibration amplitude the oscillations from periodic become chaotic were constructed. The bifurcation boundaries in the damping coefficient and vibration frequency were established as well. To improve system quality and eliminate chaotic phenomena in it, it was proposed in [78] to introduce a control that drives motion from chaotic to periodic. Various methods of control were considered such as introduction of an additional constant or periodic torque, time-delayed feedback control, adaptive control, bang-bang control, optimal control, and introduction of additional pulse action. The paper presented numerous graphs depicting the results of modeling the original and controlled systems and demonstrating applicability of the proposed methods.

3.2. Space Engineering

Chaotic spacecraft angular oscillations and their control are discussed in numerous recent publications. Besides the traditional problems of control of space structures taking into account the elastic deformations of their elements (see, for example, [9]), other problems arise where the spacecraft is considered as a rigid body and complex (including chaotic) oscillations result from nonlinearity of its dynamics.

Spinning satellite with a peripheral damper of nutation oscillations was considered in [21, 127, 128]. The system consists of a rigid body spinning around a main axis and an energy absorber in the form of a peripheral inertial spring damper. Additionally, small jets can develop a control torque about the aforementioned axis. The satellite is also subjected to a variable perturbing torque that is regarded as harmonic. In practice, such a torque may occur, for example, if the spin velocity of an out-of-balance rotor mounted on the satellite is varied. On the basis of the Lyapunov function method and heuristic considerations, [127, 128] constructed a control algorithm to stabilize the desired speed of satellite spin in the absence of nutation and precession. The same problem was considered in [21], but the control law was designed by the speed-gradient method with the use of the energy objective function. This method was shown to attain its object with a smaller level of control.

The plausibility of chaotic motion in *gyrostat* and methods of its control were studied in [77, 95, 116]. The gyrostat is a body with three rotational degrees of freedom and one or more internal wheels. Studies of gyrostat dynamics and control are of practical importance because these models describe satellites performing angular spin motion and dual-spin spacecraft such as satellites with wheel motors or spinning satellites with stabilized platform.

Analysis, control, and synchronization of the chaotic processes in a gyrostat subjected to external disturbances were carried out in [77]. Consideration was given to the dynamics of a gyrostat having three wheels with mutually orthogonal axes of rotation. The wheels are driven by electric motors. It was assumed that a small sinusoidal ripple is superposed on the rotation torque of one of the rotors. The current in the motor of one of the wheels can be varied, thus creating a control action. The state vector of the system at hand consists of the satellite angular spin rates in the axes of the vehicle state coordinates and current in the control motor. The authors of [77] believe that studies of chaotic motions in the gyrostat are of practical value, in particular, because it can be used as a missile model. Anticontrol of the missile angular chaotic motion at attack hinders intercept because in this case the trajectory of motion is hardly predictable. Like [78], this paper presents the results of using various methods for analysis of uncontrollable motion of the plant. Analysis demonstrated that angular motion of the gyrostat can become chaotic with reduction of the frequency of external disturbances. The work proposed and considered algorithms of adaptive and time-delayed feedback control to change the nature of system oscillations, that is, make motion periodic and not chaotic. Additionally, consideration was given to the possibility of anticontrol of chaos by an arbitrarily small control. With that end in view, it is proposed to use a small constant or periodic control action. The paper then studies synchronization of chaotic processes in the two aforementioned systems. Consideration was given to synchronization with linear, sinusoidal, exponential, and adaptive feedbacks. We note that the paper presents no particular treatment of the problem of synchronization for the systems under study.

Chaotic oscillations of the gyrostat with time-dependent torques of inertia and fixed position of the center of mass were considered in [95, 116]. Periodic changes of the moment of inertia were examined. As was proved by means of the Mel'nikov method, the system manifests chaotic behavior in the sense of Smale's horseshoes in the absence of external disturbances and rotation of the rotor. Stabilization is exercised by rotation of the rotor about one of the main axes of inertia of the platform.

The possibility of chaotic angular oscillations of the satellite and their suppression were studied also in [51]. Consideration was given to the motion of a satellite having constant magnetic eigenfield under simultaneous action of the terrestrial gravity and magnetic fields. For the satellite libration angle $\varphi(t)$ in the orbit plane, the following mathematical model was established under certain assumptions:

$$C\ddot{\varphi} + c\dot{\varphi} + 3\omega_c^2(B - A)\sin\varphi\cos\varphi + \mu_m i I r^{-3}(2\sin\varphi\sin\omega_c t + \cos\varphi\cos\omega_c t) = M_c(t). \qquad (7)$$

Here, c is the coefficient of proper satellite damping; ω_c is the value of satellite angular speed on the orbit; A and B are its main moments of inertia ($B > A$); μ_m is the magnetic constant; I is the value of the satellite magnetic moment; r and i are the orbit radius and inclination; and $M_c(t)$ is the value of the control torque. On the basis of the Mel'nikov method and numerical analysis, the paper proved that within some parameter domain the satellite angular motion is chaotic in the absence of control ($M_c \equiv 0$). To obtain the desired process $\varphi(t)$, the method of feedback linearization [6, Section 4.2] was used to construct the law of generation of the control torque M_c by the output and derivative. The feedback system was shown not only to suppress chaotic oscillations, but also to provide the desired form of $\varphi(t)$ (numerical examples of stabilization of the angle φ and harmonic oscillations with the given frequency were given). We note that solution seems rather trivial from the point of view of the system theory: the control torque is chosen so as to compensate the nonlinear—in the control error—term in the right-hand side of (7) and introduce proportional and differential terms.

3.3. Electrical and Electronic Systems

Chaotic processes have been recently found in many electrical and electronic devices, and methods of control were developed for them. We note that in electrical engineering conclusions about the plausibility of occurrence of chaotic processes in these systems, analysis of bifurcations, and determination of the oscillation parameters can be verified rather simply. Therefore, it is no wonder that the well-known "classical" generators of chaotic oscillations (systems of Chua, Matsumoto, Lorenz, Rëssler, and others) were embodied in electrotechnical devices. Works on control of buck converters [42], d.c. motors [48], ferroelectric systems [83], electrostatic transducers [113], d.c. transducers [66, 115, 149], power systems [165, 166], and so on must be mentioned among applications of control. Let us consider some examples.

A method of controlling the Chua system with adjustment of the adaptive gain was considered and its modification proposed in [170]. To carry out experimental studies, the authors developed an analog-to-digital laboratory model including an electronic Chua circuit with a gyrator as the inductive element and a control computer. For deviations of capacity and inductance from the rated value, respectively, by 50 % and 25 %, tracking of the control signal was established to remain acceptable. The model is used also for experiments on information transmission by modulation of the chaotic signal.

The automodulation modes arising in the backward-wave tubes (BWT) were discussed in [63]. These tubes find wide application in physical experiments in relativistic electronics and may be used to generate multifrequency, including chaotic, signals. From the standpoint of dynamics, they represent the distributed self-oscillating systems. The automodulation modes were previously studied both theoretically and experimentally in [8] for a powerful BWT with an electrodynamic system in the form of a weakly corrugated waveguide. As was established in this work, the modes of stationary generation and periodic sinusoidal and chaotic automodulation are observed in the BWT depending on the current. As was noted in [63], automodulation is desirable if it is required to obtain multimode chaotic oscillations, but its action may become unfavorable if it is required to concentrate the output power at a certain frequency. In order to suppress automodulation, the authors of [63] considered the use of a time-delayed feedback. It was suggested for this purpose to control the intensity of the input electron beam depending on the output. After rectification and filtering, the output passes through two branches into one of which a delay equal approximately to half automodulation period is fed. Then both signals are fed into a differential amplifier whose output defines the voltage bias on the control grid of the electronic gun and in that way controls the current of the BWT input beam. Computer simulation corroborates effectiveness of the proposed scheme.

An d.c. motor-based experimental facility was described in [48]. It includes electronic interrupter, generator–motor pair, and analog controller. The load torque of the motor can be controlled by the generator load current. It is required to maintain the given rotor velocity. For this purpose, the controller realizes the proportional feedback law in velocity. Theoretical analysis and experiments demonstrated that the system manifests chaotic behavior for high gain and high voltage of the rotor source of the motor. To eliminate this phenomenon, an additional time-delayed feedback was used in [48]. It was demonstrated that by varying the coefficient of this feedback under constant delay the system oscillations can be driven from chaotic to regular with period 1 (one-mode mode) or period 2 (subharmonic oscillations).

Use of the *chaotic pulse-width modulation* for elimination of dark and light stripes observable on the fluorescent lamps was studied in [117]. The Chua circuit was used as the generator of chaotic control signal. The output voltage of this circuit controls the pulse duration whose rated value is half of the period. The signal consisting of constant and chaotic components is fed into the pulse-width modulator which controls the switches in the power supply circuit. This method is superior to its existing counterparts in that it requires modifications only in the modulation circuit and not in the power one. The same paper [117] presents the results of experimental study of the proposed method.

Control of the *magnetic suspension* (levitation) was considered in [100] whose authors suggested to use the relay-logical control law where the control action (on/off switching of the electromagnet in the case at hand) depends on the prehistory, that is, on the sequence of intersections of the threshold level by the body, for which purpose the body position sensors are used. This approach allows one to do without explicit measurements of the body velocity, and it deserves mentioning that it is not a novelty for the control system designers. The model of the second order was examined analytically, and the effect of inertiality of the inductance coil was simulated on computer. As was established numerically, chaotic oscillations occur in the steady-state mode. The size of the chaotic attractor (in vertical displacement) turned out to exceed the distance between the levels of switching approximately by the factor of four. The results obtained were verified by laboratory experiments. Body displacements were sensed by an optical device, which enabled one to take the spectral response of steady oscillations. The form of response is characteristic of the chaotic processes.

Behavior of an electromechanical system consisting of interacting electrical and mechanical oscillators was considered in [177]. Both parts are coupled by the electromagnetic force developed by a permanent magnet. As the result, the Laplace force acts on the mechanical part, and the electromotive Lorenz force occurs in the electrical circuit. Models of this kind are characteristic of the *electromechanical transformers* such as loud-speakers. The aforementioned paper considered a system with a nonlinear electrical part obeying the Duffing equation. For this purpose, one makes use of the nonlinear capacitor with plate voltage V_c depending cubically on the charge q: $V_c = q/C_0 + \alpha q^3$, where C_0 is the linear part of the capacitive characteristic and the parameter α defines nonlinearity of the capacitor and depends on its type. The mechanical part is a linear oscillatory system. This transformer obeys the following equations:

$$\begin{cases} L\ddot{q} + R\dot{q} + q/C_0 + \alpha q^3 + lB\dot{z} = v_0 \cos \Omega t \\ m\ddot{z} + \rho z + kz - lB\dot{q} = 0, \end{cases}$$

where L and R are, respectively, inductance and active resistance in the electrical part; v_0 and Ω are, respectively, the amplitude and frequency of the external harmonic voltage; l is the length of the section of interaction of the magnetic field of intensity B with two moving rods to which a body of mass m is attached; k is the coefficient of spring elastic stiffness; ρ is the viscous friction coefficient; and $\dot{q}$ is the current in the electrical circuit. For the model obtained, oscillation stability was analyzed theoretically by means of the method of harmonic balance and the Floquet theory. The values of the Lyapunov exponents were also obtained, and the bifurcation diagrams reflecting the passage from regular to chaotic behavior were constructed. The problem of suppressing chaotic oscillations or reducing them to regular oscillations by a feedback controller was studied in [177]. It was suggested to make use of the vector control by deviations of the capacitor charge and movements of the body about the given values. Additional voltage in the electrical part of the system and additional force applied to the load are the components of control. Linear approximation was used in choosing the controller parameters. The passage from the chaotic mode of oscillations to the periodic mode was demonstrated numerically.

Voltage collapse in electrical generators was discussed in [84]. This phenomenon means that an inadmissibly low voltage in an appreciable part of the power grid can result from a sequence of effects caused by voltage instability. Voltage collapses recently occurred more than once in the large power grids of some countries because of load reduction that was caused by switching off many electrical customers. The authors of [84] considered a three-bus low-power grid having two generators feeding a load in the form of paralleled inductive, capacitive, and active elements. It was shown that with the increase in impedance of the first generator, the chaotic mode arises through period-doubling after the Hopf point of bifurcation. To control(stabilize, suppress) chaotic oscillations, global linearization by feedback (see [5, Section 4.2]) was used. The control system has two feedback loops, internal for linearization and external for control. The external loop includes a PI-controller. Impedance is used as the control variable. Results of comparing this method with nonlinear feedback control were presented.

Synchronization of electronic circuits with chaotic behavior was considered in [37]. Consideration was given to the circuits with the so-called *jerk equation* where the derivative of the acceleration of some variable $x(t)$ is defined. In the abstract form, this equation is as follows: $x^{(3)} = -Ax^{(2)} - \dot{x} + G(x)$, where A is a constant parameter and $G(\cdot)$ is a nonlinear function. The authors of [37] considered electrical circuits with the piecewise-linear function $G(\cdot)$ where chaotic oscillations occur. Using the linear system of second order as an example, the paper presents the concept of state observers that was used to synchronize the nonlinear circuities. Operability of this method was corroborated by numerical simulation.

3.4. Communication Systems

Extensive literature devoted to the possibility of using chaotic processes for transmission of communications allows one to state that there exists a mature area of research both in telecommunications and studies of dynamic chaos. These problems were covered in special issues of *IEEE Transactions on Circuits and Systems* and *International Journal of Circuit Theory and Applications*, reviews, and monographs [10, 12, 24, 26, 93, 94, 96, 101, 112, 178]. Three distinctive characteristics of the chaotic processes that make dynamic chaos promising for transmission of information were presented in [26].

(1) *Broadbandness*. Chaotic signals are nonperiodoc and have continuous spectra occupying, for many types of the chaotic signals, very wide bands. Moreover, one can define the form of the spectral characteristic. In the communication systems, the broadband signals are used to suppress distortions in the signal channels such as fading or narrowband disturbances. Therefore, the chaotic signals have potentialities for spread-spectrum communications.

(2) *Complexity*. Chaotic signals have complicated structure and are extremely irregular. The same chaotic generator can develop different processes in response to very small variations of the initial conditions. This fact appreciably hinders determination of the generator structure and prediction of the process for any long period. Signals of complex form and unpredictable behavior are the classical cryptographic signals, which offers one more possibility of using chaos.

(3) *Orthogonality*. Owing to irregularity of the chaotic signals, their autocorrelation function usually decays rapidly. Therefore, the signals of more than one generator can be reasonably regarded as noncorrelated orthogonal ones. This is indicative of applicability of the chaotic signals to the multiuser communication systems where the same frequency range is used simultaneously by several users.

Studies on application of chaos to the communication systems open wide opportunities in domains such as receiver–transmitter synchronization [7, 11, 53, 54, 109, 142], message masking and reconstruction [38], noise filtering [152], restoration of information signals [125], and also development of the coding–decoding algorithms allowing one to represent an arbitrary digital message through the symbolic dynamics of a chaotic system [38, 39, 125, 126].

Dynamic systems were classified in [40] from the standpoint of using them as sources of chaotic signals which carry coded information and can be transmitted to the receiver and decoded with insignificant distortions. The main fact established in this paper lies in that information can be transmitted with a very small error probability if the rate of information generation by a chaotic system, that is, the system *topological entropy*, is not smaller than the rate of information generation by the message source, that is, the *Shannon entropy*, with the deduction of the conditional entropy caused by the channel constraints such as noise distortion. The dynamic systems where the topological entropy coincides with the Shannon entropy were called by the *optimal encoders*.

Many papers were devoted to information transmission by means of the modulated chaotic signal. This method of modulation has some advantages as compared with the traditional modulation of the harmonic signal. Indeed, if in the case of harmonic signals there are only three controlled characteristics such as amplitude, phase, and frequency, then even a minor change in a parameter provides in the case of chaotic oscillations a reliably detectable change in the nature of oscillations [10], which means that the variable-parameter sources of chaos have a wide set of schemes for inserting the information signal in the chaotic signal, that is, modulating it by information. Moreover, chaotic signals are inherently wideband. The wide band of carrying signals is used in the communication systems both to increase the information transmission rate and improve system stability to disturbances. Similarity to noise and self-synchronization of the chaos-based systems give them priority to the traditional systems with extension of spectrum that are based on the pseudorandom sequences.

The monograph [12] describes various methods of information transmission using synchronization of chaotic system such as (1) chaotic masking, (2) switching of chaotic modes; (3) nonlinear mixing of the information signal with the chaotic one; (4) use of the automatic phase control; and (5) use of the methods of adaptive reception. Comparative analysis suggested that the scheme with nonlinear mixing [7] was preferable; therefore, it was chosen as the basis for research. The monograph presented diverse results of numerical studies and laboratory experiments demonstrating plausibility of using this method for information transmission. Mentioned were also the main difficulties arising in applications such as the mismatch of the elements of the transmitter and receiver that plays the principal part in wire communication and, in addition, distortions of the signal in the communication channel for wireless information transmission. The authors concluded that, though practical application of chaotic signals still faces difficulties, the era of wide application of the dynamic chaos is already on its marks.

Let us consider in more detail some schemes of using chaos for message transmission. The papers [53, 54] seem to be the first and probably the most frequently quoted publications on message transmission by means of chaotic signals. In these publications, the transmitter is constructed as a Lorenz system whose equations are scaled to the following form:

$$\begin{cases} \dot{u} = \sigma(v - u) \\ \dot{v} = ru - v - 20uw \\ \dot{w} = 5uv - bw. \end{cases} \tag{8}$$

An analog electronic circuit with the parameters $\sigma = 16$, $r = 45.6$, and $b = 4.0$ was constructed according to (8) (the variables u, v, and w corresponds to the output voltages of the operational amplifiers). The receiver equations were as follows:

$$\begin{cases} \dot{u}_s = \sigma(v_s - u_s) \\ \dot{v}_s = ru - v_s - 20uw_s \\ \dot{w}_s = 5uv_s - bw_s. \end{cases} \tag{9}$$

Equations (9) resemble (8), except for the fact the right-hand side of (9) depends not on "its" state variable u_s, but on the variable u which, therefore, can be regarded as the transmitter output arriving to the receiver. Using the method of Lyapunov functions, it was shown in [53, 54] that systems (8) and (9) are synchronized, that is, the mismatch between their corresponding state variables asymptotically goes to zero. Stated differently, (9) is an asymptotic observer of (8). The coefficient b of the transmitter (8) for transmission of the binary signal was changed to $b = 4.4$, which corresponds to the binary "unity," whereas the original value $b = 4.0$ meant the binary "zero." The level of the mismatch signal $e = u - u_s$ in system (9) grows dramatically upon change of b in (8) to $b = 4.4$ because the parameter b of observer (9) differs from b in system (8)). Averaging of $e^2(t)$ allows one to determine what signal was transmitted.

The possibility of using chaos for information protection was demonstrated in [54]. The approach proposed in this paper, which is known as "chaotic masking," lies in adding a chaotic signal to the information (useful) signal in the transmitter and restoring the useful signal from this mix in the receiver. Robustness of the process of synchronization of systems (9) and (8) was used in [54] to extract the useful signal. System (9) can be, therefore, regarded as a filter tuned, loosely speaking, in resonance to the chaotic generator (8). Since the useful signal $m(t)$ has a distinction in form from the chaotic signal, it can be restored by inputting the mixed signal $s(t) = m(t) + u(t)$ in the receiver (9) and then performing restoration by the formula $\widehat{m}(t) = s(t) - u_r(t)$ to the estimate $u_r(t)$ of the variable $u(t)$.

Synchronization-based message restoration in the receiver is allied to estimation of the plant state (here, transmitter) by observer (here, receiver) with which the control theory is well familiar. This approach gave rise to a line of research known as the observer-based approach to chaotic synchronization [28, 121, 132, 133]. In distinction from the well-known problem of estimation of linear system state, when designing observers of chaotic processes, one encounters fundamentally nonlinear signal sources giving rise to substantial difficulties [121, 133].

A more complicated, but potentially more flexible method of message restoration from the modulated chaotic signal is based on adaptation—in particular, on using adaptive observers. The paper [71] considered synchronization of two nonlinear systems (here, receiver and transmitter) in the conditions of incomplete measurements and incomplete information about the source parameters. The modulated (information) parameters are introduced linearly in the model.

The controlled chaotic modulated-signal generator obeys the Lur'e equations

$$\dot{x}_d = Ax_d + \varphi_0(y_d) + B\sum_{i=1}^{m}\theta_i\varphi_i(y_d), \quad y_d = Cx_d, \tag{10}$$

where $x_d \in \mathbb{R}^n$ is the vector of modulator state, $y_d \in \mathbb{R}^l$ is the vector of output (transmitted signals), and $\theta = [\theta_1, \ldots, \theta_m]^{\mathrm{T}}$ is the modulator parameter vector carrying information about the transmitted message. Design of a receiver (*demodulator*) proceed from the assumption that the nonlinearities $\varphi_i(\cdot)$, $i = 0, 1, \ldots, m$, the matrices A and C, and the vector B are known.

A passification-based demodulator [71] is a variety of the adaptive observer which, for known A, B, and C, obeys the equations

$$\begin{gathered}\dot{x} = Ax + \varphi_0(y_r) + B\left(\sum_{i=1}^{m}\hat{\theta}_i\varphi_i(y_r) + \hat{\theta}_0 G(y_r - y)\right), \\ y = Cx,\end{gathered} \tag{11}$$

$$\dot{\hat{\theta}}_i = \psi_i(y_r, y), \quad i = 0, 1, 2, \ldots, m, \tag{12}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^l$, $\hat{\theta}_0 \in \mathbb{R}$, $[\hat{\theta}_1(t), \ldots, \hat{\theta}_m(t)]^{\mathrm{T}}$ is the vector of parameter estimates, and $G \in \mathbb{R}^l$ is the vector of weight coefficients. The adaptation algorithm (12) which is obtained by a routine use of the speed-gradient method [15] is as follows:

$$\dot{\hat{\theta}}_i = -\gamma_i(y - y_r)\varphi_i(y_r), \quad i = 1, \ldots, m, \quad \dot{\hat{\theta}}_0 = -\gamma_0(y - y_r)^2, \tag{13}$$

where γ_i $(i = 0, 1, \ldots, m)$ are the positive gains of the algorithm. In the presence of noise in the communication channel, algorithm (13) may be regularized (robustified) by introducing a parametric feedback or deadzone.

The properties of algorithm (13) and the model examples of message transmission by the controlled Chua system can be found in [4, 5, 31, 32, 71, 72]. Development of this approach which is related with extension of the methods of design of adaptive observers to the nonpassifiable systems was presented in [73] where two methods of designing adaptive observers were proposed and the possibility of using them for message transmission was demonstrated. The method of Lyapunov functions and the Yakubovich–Kalman lemma were used in [67] to design the adaptive observer. Illustrative numerical examples of information transmission by modulation of the parameters of the Lorenz and Chua systems were given. Another approach based on identification of the transmitter parameters from a discrete model can be found in [34].

The issues of robust synchronization of chaotic systems and its possible use for information transmission were discussed in [68]. It was noted that their study is important for ascertaining, on the one hand, the security level of message transmission and, on the other hand, the possibility of restoring the information signal at the mismatch between the real and calculated models of the communication system devices. State observers are used in [68] as above.

Use of the methods of control of chaotic processes for message coding in communication channel was considered in [39, 126]. This coding lies in deliberate redundantization of the transmitted message so as to enable the receiver to restore the message and correct the errors occurring in the channel. We note that by the *control* is meant here the possibility of acting upon the system so that a sequence of characters carrying the message is generated instead of a pseudorandom sequence generated by the uncontrollable system. At that, the system must behave chaotically on the whole. An example of realization of a communication system with chaotic carrier can be found in [25].

Some studies were devoted to the use of the optical (laser) channel for message transmission. For example, [17] discussed the possibility of constructing an optical information channel based on two synchronous CO_2 lasers with periodic pumping of the active medium. For frequencies of pumping generation that lie near the frequency of system relaxation, the laser generates chaotic pulses with information entered by amplitude modulation. The second (receiving) laser is controlled by injection of radiation from the transmitting laser. The output radiation is used to restore the modulating signal. The experimental setup has two microchip Nd:YVO_4 lasers with periodic pumping by laser diodes. An acoustooptic modulator with the modulation frequency 4 MHz and depth less than 0.2% was used for external modulation of the laser output radiation. Modulation of the output radiation corresponds to transmission of unit, no modulation corresponds to zero. The frequency of variations of the binary signal in experiments was 100 kHz. As above, synchronization of the chaotic processes in the transmitting and receiving lasers was used for message decoding. The experimental data given in [171] are indicative of a satisfactory reproduction of the information signal at the receiver output. An experimental setup for transmission of information by synchronization of two chaotic lasers can also be found in [168]. It makes use of two semiconductor lasers of the 1.3μm range embraced by the optoelectronic time-delayed feedback through high-speed InGaAs photodetectors with the 6 GHz pass band. Passage from regular through quasiperiodic to chaotic oscillations of radiation intensity was observed at reduction of the delay time from 7.47 nsec to 6.9 nsec. The transmitter light flow was sensed by a photodetector whose output was summed with the receiver feedback. In the experiments on information transmission, the useful signal (message) was added to the transmitted feedback. The message was restored at the receiver output using the method of [54] by subtracting the signal of the receiver chaotic generator from its input. Good restoration of the binary signal represented by the pulses of small on-off ratio with the repetition rate 100 MHz was confirmed experimentally. Synchronization of the chaotic semiconductor lasers and the possibility of using it in the communication systems were discussed also in [143] which presented the experimental data on measuring the relative cavity phase of the receiver and transmitter. The results obtained show that this parameter bears on the quality of synchronization.

3.5. Information Systems

Various methods of using the chaotic processes for storing and coding information have been proposed by now. It was noted in [3] that basically new information processing systems, chaotic processors, appear in the distance. The potentialities of these processors were demonstrated by "Associative Memory for Pictures" software system intended to recode and access images and the "FacsData Wizard" for control of facsimile documents [3]. The "Nezabudka" (*forget-me-not*) software system [3, 13] protected by Russian (RF2050072) and USA [62] patents is an extension of this system. It is oriented to accessing documents and finding there certain places in response to natural-language requests. Information is recorded and stored in the form of trajectories of a discrete chaotic system. The corresponding chaotic map is constructed in the course of information coding. Upon starting under arbitrary initial conditions, the trajectory after a transient process is attracted to one of the available cycles and reproduces the corresponding information.

Design of an associative memory based on parametrically related chaotic elements was discussed in [97]. Experimental data on storing information in a chaotic system with a time-delayed feedback were presented in [122], and [114] was devoted to transient chaos-based coding of binary data.

3.6. Chemical Industry and Raw Material Processing

Chaotic stirring, especially of liquids and free-flowing materials, is an important area of studies on the use of controlled chaos. Good stirring of flows, which must be maintained in the combustion chambers, heat exchangers, and other installations [135, 159], is of applied importance in chemical industry for continuously stirring reactors, production of powders and polymers.

In chemical industry, higher rate and quality of stirring reduce the mass of reagents that are not involved in reaction and, consequently, improve the product. As was noted, the cost of purification makes about 80 % of the cost of the finished product. Therefore, careful stirring is of prime importance here. A method for designing control to stabilize chaotic nonisothermal processors in the continuous stirred tank reactors under constrained amplitude and rate of the control action was proposed in [175]. The authors of [99] used an extended variant of the Pyragas algorithm to control the bubbling gas-solid fluidized bed reactors. This control diminishes the bubbles, which in turn leads to better mass transfer of the reagent gas through the bubbles to the catalyst particles.

Stirring of liquid flows. A general method of improving the stirring rate of chaotic flows of liquids on the basis of increased *degree of chaoticity* was proposed in [159]. This degree is determined using the local values of the Lyapunov exponents characterizing the mean increase in the phase volume. They describe motion of liquid particles in terms of the Hamilton equations on the basis of Lagrange description of the dynamics of two-dimensional flows. Then, consideration is given to the general equations of the following nonlinear dynamic system:

$$\dot{x} = F(x(t), u(t)), \tag{14}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t)$ is the flow control parameter. Equations (14) are considered together with the variational system

$$\dot{w} = M(x, u)w, \tag{15}$$

where $M = \partial F/\partial x(x, u)$ is the Jacobi matrix. It was proposed in [159] to measure the local velocity of flow expansion through the quadratic norm w as $\frac{\partial}{\partial t}|w|^2 = 2w^{\mathrm{T}}M(x, u)w$ and to vary the control parameter according to

$$\Delta u = \gamma \operatorname{sgn}(w^{\mathrm{T}}\frac{\partial M}{\partial u}w), \tag{16}$$

where $\gamma > 0$. Control action is activated where the greatest local Lyapunov exponent is smaller than its mean value. One can easily see that algorithm (16) is a special case of the speed-gradient algorithm (see [6, Section 4.2]) using the objective function $Q(x) = |x|^2$. A discrete control algorithm which also is a special case of the gradient algorithm of [6, Section 4] also is presented in [159] where it is used to improve the degree of chaoticity of the Chirikov (standard) map. The disadvantage of this method lies in the assumption that it is possible to measure the entire state vector and that the parameters of the controlled process are known.

Stirring of free-flowing materials. Spontaneous chaotic granular stirring which is observed in simple cylindrical drums filled in part by small granules was described in [160]. This phenomenon may be observed for sufficiently small—at most 300 μm in diameter—granules. It is assumed that the mechanism of appearance of chaotic motions is as follows: the periodic stick-slip motion arises upon shifting the layer separating the fixed and flowing granular zones, which, upon stirring, leads to much weaker adhesion of the granules as compared with that described by the same authors in earlier papers [102, 129].

Stirring of free-flowing materials leads to occurrence of both many nice pictures and examples of self-organization (generation of structures) [87]. One must bear in mind that more active stirring by forceful shaking or faster tumbling does not guarantee a better final mixture. The point is that it is inherent in the free-flowing materials to be divided in density and size of particles by shaking, so that the forced stirring can lead to an opposite result. Self-organization occurs on the basis of two competing phenomena: chaotic advection or chaotic stirring as in liquids, and separation which has no counterpart in liquids. These systems represent the simplest kind of systems where chaos and self-organization are observed and which can be subjected to laboratory studies. The readers are referred to [87] for a good description of stirring of free-flowing materials.

Many authors carried out numerical or experimental study of the quality of stirring as a function of various parameters. As was shown in [103], the flow in elliptical or square mixers, in contrast to the circular mixers, is time-periodic, which leads to chaotic advection and accelerated stirring. As for generation of the external action, the majority of authors accept the open-loop control in the form of periodic time function. This action was used to improve stirring of free-flowing materials [45]. Stirring of liquids in the two-dimensional square cavity under pulsing periodic velocity of the cover was considered in [30] by means of spatial discretization of the spectral element. It seems that the most strict treatment of the problem of optimal stirring was given in [56] whose authors consider the prototypical problem of stirring as that of control aimed at determining a mode of liquid flow variation such that the entropy is maximized. To this end, a corresponding apparatus of the ergodic theory is used to determine the entropy of the periodic sequences and the mode of operation maximizing the entropy among all periodic modes generated by the mutually orthogonal flows.

The idea of chaotization of stirring of free-flowing materials was introduced into industrial practice in the machines manufactured by Kroosher Technologies Company,[3] whose products include a "Kroosher" mechanical contrivance which is attached to the rod of a vibrating screen and creates additional oscillations imparted to the machine actuator by means of wear-resistant internal mechanical parts. As the result, the energy of one-frequency oscillations is rearranged between the frequencies in a wide range. Multi-frequency excitation is amplified on the screen cells owing to the resonance characteristics, which improves the installation productivity.

4. OTHER APPLICATIONS

Scheduling of industrial processes (in the wide sense of the word) was discussed in [151] which considered a system consisting of concurrently operating machines and a distribution switch (*server*). Depending on the properties of the system under study, a mode with chaotic switchings of the server may arise. The paper calculated the probabilities of distribution of the chaotic return times and analyzed losses of products caused by switchings vs. the maintenance schedule. The paper proposed and substantiated a maintenance schedule minimizing the general losses of switching time, which improves productivity of the entire system. Consideration was also given to the production lines that usually have more than one level of interacting machines. Data on modeling a three-level production system with neighboring switching instants were given. Numerical analysis revealed occurrence of chaotic "travelling waves." The results of modeling also demonstrated that individual machines are not synchronized, but "global" synchronization of the switching frequencies is observed between the serially organized network levels.

The number of existing and potential applications of chaos and its control is rapidly growing. The following of the recent applications deserve mentioning:

(1) applications to the numerical methods of analysis (stability analysis of fixed points [123] and stabilization of the Richardson eigenvector algorithm [85]);

(2) use of chaos for information processing [58, 108, 161];

(3) control of system complexity [69].

[3] Company site http://www.kroosh.com/

5. CONCLUSIONS

Separation of a wide class of control-theoretical applications (cybernetics in a wide sense of this word) oriented to the development of other scientific theories, rather than to industrial applications is an important feature of the state-of-the-art in cybernetics. The majority of such works are published in physical, rather than engineering, journals, which is indicative of the advent of a new branch of physics, the cybernetical physics [20, 22] which applies cybernetical analysis to physical systems. Control of chaos is a branch of cybernetical physics. Complexity of the chaotic dynamics gives rise to new problems of control that stimulate further development of the control theory.

An important class of applications relies on chaos to describe uncertainty in the behavior of dynamic systems. In contrast to other methods of describing uncertainty such as stochastic, fuzzy, and so on, the chaotic system provides a natural tool for describing uncertainty of oscillation characteristics (frequency, phase, amplitude) by a few parameters. Development of the methods of control under chaotic uncertainty was started only recently.

It deserves noting that complexity of the arising problems compels the researchers to use simplistic models of the control plants even in the works dealing with technical applications. In fact, many applications are just "potentialities" whose realization is possible only in future and only under certain conditions. Nevertheless, the present authors believe that familiarization with new formulations and their discussion can be beneficial for future development both of the control theory and its applications.

ERRATA

The present authors take an opportunity to point out the errors committed in Part I of the present review (B.R. Andrievskii and A.L. Fradkov, Control of Chaos: Methods and Applications. I. Methods, *Avtom. Telemekh.*, 2003, no. 5, pp. 3–45):

1. p. 695, line 7 above: read "matrix $\Phi(\tau)$" instead of "matrix $\Phi(t)$"
2. p. 684, line 27 above: read "$\lambda > 0$ and for all" instead of "λ and for all"
3. p. 685 after formula (31): read "such that the closed" instead of "if closed"

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